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Coherence conditions for the forward scattering of neutrinos

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Abstract: We rederive the refractive effects of neutrinos as the interference of coherent scatterings. We study the details of this approach, including the features of wave packets, the possible effects of the recoil of the scatterers, the thermal motion and the polarisation of the matter. Our calculations demonstrate how the coherence is lost whenever there is an observable change in the scatterer state. Particularly, flavor changing dispersive effects are found to be smaller than previously claimed. For a degenerate medium the scattering model suggests a suppression of the refractive effects.

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1. Introduction

Neutrinos interact utmost weakly with matter, and can thus pass through almost anything without any observable effect — almost. It has been proposed [1, 2] (first in the context of the detection of relic neutrinos [3]) that in certain environments there could be enough interaction to cause a smallest effect to the propagation: coherent forward scattering. This forward scattering causes effect analogous to refraction of light in matter, although in a much much weaker scale. Thus observing any direct effects of refraction (not to even think about reflection) is, of course, implausible [4].

Indirectly the refraction modifies the neutrino flavour oscillations [5] by inducing small frequency shifts between different neutrino flavours (somewhat but not fully analogous to birefringence, optical activity or chirality in optics). These phase shifts could then enhance transitions between neutrino flavors by a resonant conversion [6].

The resonant neutrino conversion, or MSW-mechanism, is considered as a plausible solution to the contradiction between the predicted [7] and measured [8] solar neutrino fluxes. The matter effect has been found also to be very important for neutrinos inside collapsing stars [9], neutrinos passing through earth, and neutrinos in the early universe [10].

Macroscopically, the refraction can be conveniently modeled introducing a refractive index, exactly as in optics. Here the refractive index is actually a matrix in flavour space; 3×3 for three flavours, 6×6 if right-handed neutrinos enter the game. Of course, this macroscopic quantity must be bound to microphysics to find reasonable values for it, and to find the limits of validity and other restrictions.

The matter effect of neutrinos is analogous to the refraction of light in a transparent medium. Other particle physics contexts where the refractive effects are effective involve the propagation of low energy neutrons through crystals (see e.g. [11]), and the regeneration of the neutral kaons (see [12]). The former resembles the photon behavior in most respects while the latter is more analogous to neutrino matter oscillations, resulting in the transformation of a pure mass eigenstate to another mixed state when passing through a bulk of matter. The generation of the other kaon state due to the matter effect has been verified experimentally, and the result agrees with the model of the matter effect caused by the coherent forward scattering.

The matter effect is usually treated as a semi-classical effective potential determined by the average distribution of the background matter fields. Ref. [13] rederived the result using a microscopic scattering model, but restrained themselves to one dimension. Ref. [14] applied the usual optical derivation of the refractive index. One has also applied the finite temperature field theory [15] which has the advantage of incorporating nicely the effects of the temperature.

In this article we study the refractive effects of neutrinos starting from the scattering picture. This approach provides a very clear and physical way to understand the matter effect. We assign the matter effect as the coherent interference of the individual scattering processes between propagating neutrinos and the constituents of the medium. The interactions are treated perturbatively using the Born approximation. We intend to clarify the effects of discrete medium, inelasticity, the recoil of the scatterer, the scattering from moving targets, degenerate medium and the finite size of the wave packets. We pay special attention to the validity of the assumptions.

The modeling of the matter effect relies on the modeling of the medium. To be most rigorous one should treat all the particles interacting with the propagating neutrinos as part of the quantum mechanical system. We do not try this here but we intend to find the physical result to a sufficiently good accuracy using a more reduced system. We try to give a proper justification for the simplification we will made.

2. Neutrinos in matter

Traditionally the effect of the medium on the propagation of neutrinos is described by introducing an effective macroscopic potential $V(\vec{x})$, so that one could write the effective Hamilton operator

$$\mathcal{H}(\vec{x}) = p + \frac{m^2}{2p} + V(\vec{x})\,. \tag{2.1}$$

Using a macroscopic potential to describe the microscopic interactions is adequate (and formally exact) when the individual scattering processes are strictly elastic and coherent, so that there is no change in the state of the background [11].

Heuristically the potential can be extracted directly from the interaction terms in the lagrangean. In the contact approximation the interaction terms can be written, after the appropriate Fierz transformations, as

$$\frac{G_F}{\sqrt{2}}\bar{\nu}(x)\gamma_\mu(1-\gamma_5)\nu(x)j^\mu(x)\,. \tag{2.2}$$

where the weak current $j_\mu(x)$ gets contributions from both charged and neutral current interactions with all relevant particles. As the most simplest approach, assuming that the matter is isotropic and at rest, one just takes the average over matter, $\langle j_0(x)\rangle$.

A simple average over matter can be motivated by the behavior in one dimension. In that case the induced phase of a one component wave function can be obtained from the equation

$$\theta(x) = \theta(0) + \int_0^x V(x')\mathrm{d}x'\,. \tag{2.3}$$

This result does not yet tell anything about the form of the wave, neither is it valid for many component waves. For many-component waves the averaging is generally considered correct if the inhomogeneities are essentially smaller than the length scale associated to transformations among the wave components [16, 17].

In three dimensions the simple average is no more exact. Geometrical effects do have significance. Particularly, one may bear in mind that the weak interaction range is much shorter than the average distance between scatterers or the de Broglie wave length of neutrinos.

In the finite temperature field theory the effects of the background are treated implicitly by modified Feynman rules. This is a very elegant way to obtain the self-energy of the propagating particle [18], but bears the risk of hiding the physics behind axiomatic framework. The results [15] are the same as in the potential model.

The origin of the matter effect can be understood more intuitively with scattering theory. In optics the refractive index is usually derived as an interference effect of waves scattered from distinct scatterers. Hence the resulting refractive index is proportional to the forward scattering amplitude. Applying this result as is to neutrinos, one obtains the effective potentials

$$V = 2\pi \frac{\sum f(0)_i N}{k}, \tag{2.4}$$

where N is the number density of scatterers and $f(0)$ is the forward scattering amplitude (sum of different components). In this work we adopt the picture of assigning the matter effect as a result of the interference of the scatterings.

3. Coherent momentum conserving forward scattering

Let us consider first a plane wave propagating in z-direction, reaching perpendicularly an infinitely wide thin layer (at $z = 0$) of point scatterers. We assume consistently below that the scattering is always so weak that only coherent parts are observable. The observer behind the layer sees now a wave consisting of the original wave, and waves scattered off all the scatterers. Assume now that the scattering is completely elastic, and there is no momentum interchange, so that the wave at the observer ($rk \gg 1$) is given by

$$\psi(z) = e^{ikz} + \sum_i \frac{f(\theta_i)}{r_i} e^{ikr_i}, \tag{3.1}$$

where $\cos\theta_i = z/r_i$ and $r_i = \sqrt{z^2 + (y - y_i)^2 + (x - x_i)^2}$.

If we further assume scatterer distance short enough ($z \gg ka^2$), or take an ensemble average over different medium configurations and incoming waves, or just integrate over sufficiently large detector size and detection time, only requiring the scatterers to be randomly positioned, we can change the sum to an integral:

$$\psi(z) = e^{ikz}\left(1 + \int dr \int d\phi f(\theta(r,\phi)) N(z,r,\phi) e^{ikr}\right), \tag{3.2}$$

where N is the classical number density of scatterers.

Assuming an implicit vanishing in the infinity ($N \to 0$ as $r \to \infty$), the integral (3.2) is readily calculated, resulting in

$$\psi(z) = e^{ikz}\left(1 + i\frac{2\pi f(0)}{a^2 k}\right) + \mathcal{O}\left(\frac{1}{kz}\right). \tag{3.3}$$

Not so surprisingly the problem has reduced to a one-dimensional case.

If we continue assuming there are no dependencies between scatterers (no clustering, density variations, heat waves, holes etc.), and still no significant incoherent contribution, we can easily generalize the above result to finitely thick layers. With these assumptions, the wave penetrating a sample consisting of α such layers can be expressed as

$$\psi(z) = e^{ikz}\left(1 + i\frac{2\pi f(0)}{ka^2}\right)^{\alpha}. \tag{3.4}$$

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At the limit $\alpha \to \infty$ one obtains, from the product form of the exponent function,

$$\psi(z) = e^{iknz}, \tag{3.5}$$

where the index of refraction n is given by

$$n = 1 + \frac{2\pi N f(0)}{k^2}, \tag{3.6}$$

where a minor assumption of $n \approx 1$ was done to drop one cumbersome term. The relevant values of the refractive index range from $n - 1 \propto 10^{-20}$ in the Sun to $n - 1 \propto 10^{-6}$ in supernova cores.

The above procedure requires the waves being coherent. Actually, it is not enough that the neutrino wave functions add coherently, also the scatterers must keep coherent. By coherence we mean that the wave describing the scattered state (from any scatterer) must be coherent with the unscattered (incident) wave. We do not require the scatterers to be coherent with each others.

The requirement of coherence for the scatterers can be easily understood physically: the loss of coherence implies the possibility to observe the individual scattering. In case of a discrete quantum number the coherence means that the scatterer does not change its state. If the scatterer is excited to a state orthogonal to the initial state, the process does not contribute to the refraction index. For a scatterer in a mixed state the coherence is proportional to the common terms (inner product). Hence the flavor interchange processes considered in ref. [19] cannot contribute to the refraction. The case of continuous quantum numbers is discussed in the next sections.

4. Neutrinos and electrons as wave packets of finite size

We assumed above that the wave is an infinitely wide plane wave. In order to learn more about the scattering phenomena, we will next introduce a more realistic description of neutrinos using wave packets of finite size. For simplicity we assume the neutrino wave as gaussian (the results are not expected to depend a lot on the particular shape anyway),

$$\Psi_\nu(\mathbf{r}, t) = \frac{1}{\sqrt{\pi^{3/2} b^2 d}} \exp\left(ikz - iEt - \frac{(z - vt)^2}{2d^2} - \frac{x^2 + y^2}{2b^2} \right), \tag{4.1}$$

where d and b denote the length and width of the wave, and we selected the neutrino wave to propagate along z-axis, with the group velocity $v \equiv k/E$. The phase velocity is the inverse of the group velocity, $v_p = Ec^2/k$.

We have ignored the spreading of the wave packet as insignificant here. For very narrow or short wave packets a correction must be applied, otherwise e.g. integral (3.2) would cause spurious suppression factors when the scattered waves spread but the original one not. But for typical neutrino wavelength less than 1 pm already a 100 μm wide packet is wide enough to allow the above approximation, and the widths in general are expected to be much larger.

The scatterer is assumed to be in an initial state

$$\Psi_{0s}(\mathbf{r}_s, t) = \frac{1}{(\sqrt{\pi} d_s)^{3/2}} \exp\left(-iMt - \frac{r_s^2}{2d_s^2}\right), \tag{4.2}$$

where M is the mass of the scatterer, taking for a moment the scatterer to be at rest.

Many–scatterer wave function is yet assumed to be a product of single–scatterer wave functions without any interchange terms, at least not contributing between any subsequent interactions

$$\Psi_{0s}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N, t) = \Psi_{0s}(\mathbf{r}_1, t)\Psi_{0s}(\mathbf{r}_2, t) \cdots \Psi_{0s}(\mathbf{r}_N, t) \,. \tag{4.3}$$

5. Interchange of momentum in the scattering

Until this we assumed that the neutrino momentum is strictly conserved in the scattering process. In fact, the conservation of energy and momentum was a necessary requirement to obtain the result. In the following we will consider the role of momentum interchange more accurately, without making any assumption for its negligibility. We will consider also explicitly some effects of inelasticity. Here we note that there are two ways to define the elasticity; we adopt the narrow interpretation, so that we describe the scattering as elastic if the internal states of all participating states are conserved, irrespective of kinematical effects.

We assume that the scattering preserves the form of the waves (in the longitudinal direction), so that asymptotically ($t \gg 0$) we have

$$\Psi(\mathbf{r}_\nu, \mathbf{r}_s, t) = \Psi_0(\mathbf{r}_\nu, \mathbf{r}_s, t) + \Psi_S(\mathbf{r}_\nu, \mathbf{r}_s, t) \,, \tag{5.1}$$

where

$$\begin{aligned}\Psi_S(\mathbf{r}_\nu, \mathbf{r}_s, t) = {} & \frac{1}{r\sqrt{\pi^3 d b^2 d_s^3}} f_{12} \exp\left(ik' r_\nu - iE'_\nu t - \frac{(r'_\nu - v'_\nu t)^2}{2d^2}\right) \times \\ & \times \exp\left(ip'_s r_s - iE'_s t - \frac{(r_s - v'_s t)^2}{2d_s^2}\right), \end{aligned} \tag{5.2}$$

where $r = \sqrt{\mathbf{r}_\nu^2 - \mathbf{r}_s^2}$. A key assumption here is that the scattering is strictly a two body problem. Because of the very short range of weak interaction, this is quite a good assumption under normal conditions.

For the coherence effect the relevant quantity is the phase difference between Ψ_0 and Ψ_S. Since we are finally interested in the probabilities, we can actually study the term $\Psi_S^\dagger\Psi_0$, which we call the coherence term, for brevity. Note that the time dependent phase factor in the coherence term can be written as $\exp(iE_T t)$, where E_T is the total energy, conserved in the scattering. Thus apparently the time enters only through the gaussian factors.

If the scattering is elastic, i.e. there is no change in the internal states, both waves conserve their momenta in the center of mass frame of reference. In any other set of coordinates, however, this is no longer true, and the collision appears as inelastic. For the scatterer initially at rest, the final momentum of the scattered particle is obtained from the Compton formula

$$k(\theta) = k_\nu \frac{M}{M + k_\nu(1-\cos\theta)} \approx k_\nu \left(1 - \frac{k_\nu}{2M}\theta^2\right), \tag{5.3}$$

where k_ν is the initial momentum of the neutrino, θ is the scattering angle.

Whether the recoil of the scatterer can be ignored depends now on the ratio of the energy of the neutrino and the mass of the scatteree. In the limit $M \to \infty$ the scatterer can be considered as fixed, with no recoil. However, the recoil effect will be non-trivial when $k_\nu > M$. The situation is more complex, if the scatterer is not free like in ideal gas, but interacts with its neighbours in such a way that the recoil is immediately distributed to its companions. One can approximate such a medium here by introducing a concept of effective mass. In most extreme cases, such as solid crystal, this is the mass of the whole crystal.

We next integrate over the uninteresting scatterer wave function, obtaining the result

$$\begin{aligned}\int \Psi_S^\dagger(\mathbf{r}_\nu, \mathbf{r}_s, t)\Psi_0(\mathbf{r}_\nu, \mathbf{r}_s, t) d\mathbf{r}_s = \exp\left(-\frac{v_s^2 t^2}{4d_s^2} - \frac{1}{4}p^2 d_s^2\right) \times \\ \times \exp\left(-i\frac{p v_s t}{2}\right)\psi(\mathbf{r}_\nu)\,.\end{aligned} \tag{5.4}$$

The time dependent factor of the real part in the above exponent (5.4) describes the separation of the recoiled wave packet from the unturbed one. The separation of the wave packets would cease the coherence, corresponding to the physical possibility to observe the scattering. Evidently the process is coherent as long as $t < d/v$. Using for the final velocity of the scatterer v its characteristic value in the edge of the coherence cone, and taking the most extreme assumption of independent scatterers with the sizes of their wave packets being of the order of the typical interparticle spacing, the typical coherence times vary from 10^{-6} s to few seconds. This is much larger than the typical collision time among the scatterers, so that the disturbance fades away to unobservable.

Let us then consider the phase factors. Applying the definition of the group velocity, $v_s \simeq p/M = 2E_K/p$ where $E_K = E_s' - M$ is the final kinetic energy of the scatterer, and using the conservation of energy, $E_s' - M = \Delta E_\nu$, the imaginary part of the equation (5.4) can be written as

$$\exp\left(-i\frac{pv_s t}{2}\right) = \exp(i\Delta E t)\,. \tag{5.5}$$

Hence the remaining wave function reacquires the time dependent phase. Taking into account the change of momentum, the *phase difference* between the incident and the scattered wave (as appearing in eq. (3.2)) is now given by

$$\delta\varphi = kr - k'z - Et + E't\,. \tag{5.6}$$

For small angles it reads

$$\delta\varphi = \frac{1}{2}k'z\theta^2 + \delta k\left(z - \left(1 - \frac{m^2}{2k^2}\right)t\right). \tag{5.7}$$

Next we integrate over time, at a given point z. Taking carefully into account the effects of a small neutrino mass to the group velocity, we obtain

$$\delta\varphi = z\left[\frac{1}{2}k'\theta^2 + \frac{1}{4}\delta k\theta^2 + \frac{m^2}{k^3}(\delta k)^2\right]. \tag{5.8}$$

Since δk is of second order in θ, according to the Compton formula (5.3), the corrections to the phase are of the fourth order in θ. Hence for matter at rest the corrections due to the loss of momentum can be ignored in all conceivable conditions.

6. Thermally moving targets

In physical systems the scatterers are not at rest but move randomly. In this section we intend to clarify the effects of the finite temperature to the scattering amplitudes. We will also show that it has little effect on momentum interchange effects.

Previously we assumed the scattering amplitudes to be independent on energy. This approximation stems from the treatment of the weak interaction by a Fermi contact interaction. Albeit small, the internal momentum transfer (q) may be significant. We will account for it by taking the next terms in the propagator expansion,

$$\frac{1}{q^2 - M^2} \simeq -\frac{1}{M^2}\left(1 + \frac{q^2}{M^2}\right). \tag{6.1}$$

Now, for s-channel scattering we can use $q^2 = 2E_\nu E_s$, where E_s is the scatterer energy, assuming all particles to be relativistic. (Non relativistic case has no physical relevance). For t-channel reaction, however, we have $q^2 = -2E_\nu E_s$, i.e. the sign is different. Hence, for electron neutrinos the refractive contribution from electrons (t-channel) will be of different absolute value than the contribution of positrons which is due to s-channel scattering.

Since the energy of the scatterer appears linearly, for a medium in thermal equilibrium it can be replaced by the average energy given by the thermal distribution. Hence the scattering amplitudes should be multiplied by the temperature correction factors

$$1 \pm \frac{2E_\nu \langle E_s \rangle}{M} \simeq 1 \pm \frac{6.3 E_\nu T}{M}\,, \tag{6.2}$$

where $M = M_W$ for charged currents, and $M = M_Z$ for neutral currents. This coincides with the result of the finite temperature field theory. For the applications of the early universe the effective potential in thermal background is usually written in the form

$$V = \sqrt{2} G_F N_\gamma \left(N_{\mathrm{as}} - A \frac{kT}{M_{\mathrm{W}}^2} \right), \tag{6.3}$$

where N_γ is the photon number density, and N_{as} is the particle-antiparticle asymmetry of the medium, normalized to photon density, and $A \simeq 55$.

Consider now the effects of the movement of the scatterer on the momentum interchange in the scattering process. We assume that the scatterer moves with an arbitrary velocity to an arbitrary direction. We decompose its movement to the direction parallel to the incoming neutrino, and that perpendicular to it (in the scattering plane). The respective momentum components are denoted as $p_\parallel$ and $p_\perp$. The Compton formula should now be modified to

$$k(\theta) = k_\nu \left(1 + \frac{p_\perp \sin\theta - (p_\parallel + k_\nu)(1 - \cos\theta)}{E + k_\nu (1 - \cos\theta) - p_\parallel \cos\theta - p_\perp \sin\theta} \right). \tag{6.4}$$

Here E is the total energy of the scatterer. Usually it is sufficient to use

$$k(\theta) = k_\nu \left(1 + \frac{p_\perp \theta - \frac{1}{2}(p_\parallel + k_\nu)\theta^2}{M} \right), \tag{6.5}$$

for non-relativistic scatterers. For relativistic scatterers it can be most conveniently written in the form

$$k(\theta) = k_\nu \left(1 + \frac{\sin\alpha}{1 - \cos\alpha} \theta \right), \tag{6.6}$$

where α is the angle between the original scatterer momentum and the neutrino momentum, in the scattering plane. For very small α additional terms ensure that is is finite and converges to zero.

The phase difference between the incident wave and the wave scattered from the scatterer i, obtained by inserting (6.5) and (6.6) to (5.8), is given by

$$\delta\varphi = \frac{1}{2} k z \theta_i^2 \left(1 + \frac{p_{\perp i}^2 m^2}{M_i^2 k^2} \right) + \frac{p_{\perp i}}{4 M_i} k z \theta_i^3 \tag{6.7}$$

for non-relativistic scatterers, and

$$\delta\varphi = \frac{1}{2} k z \theta_i^2 \left(1 + \frac{m^2}{k^2} \sin\alpha_i \right) + \frac{1}{4} \sin^2 \alpha_i k z \theta_i^3 \tag{6.8}$$

for relativistic scatterers. The additional terms describing the irregular behavior are ignorable for all light neutrinos. They may spoil the coherence only for non-relativistic neutrinos in relativistic background. For the typical situations in neutrino astrophysics it is always sufficient to use the conventional formulas.

7. Non plane parallel medium

Above we have assumed the medium to be plane parallel (=layerwise homogeneous). Nature seldom is idealistic, and thus we study what limits this assumption poses us, without trying to bury us too deeply to this topic.

First we consider small scale fluctuations. Divide the scatterer density to homogeneous and fluctuating part $N(\mathbf{r}) = N_1(z) + \delta N(\mathbf{r})$. For the homogeneous part we have a solution $\Psi_1(z) = \exp(-ik \int dz n_1(z))$. The perturbative solution for inhomogeneities is apparantely

$$\begin{aligned}\Psi(\mathbf{r}) = \Psi_1(z)\Bigg(1 &+ \int d\mathbf{r_1}\delta N(\mathbf{r_1}) e^{-ik(1-1/\mu_1)\int ds_1 n_1(z(s_1))} \frac{f}{|\mathbf{r}_1 - \mathbf{r}|} + \\ &+ \int d\mathbf{r_1} \int d\mathbf{r_2} e^{-ik(1-\frac{1}{\mu_1})\int ds_1 n_1(z(s_1)) - ik\left(1-\frac{1}{\mu_2}\int ds_2 n_1(z(s_2)\right)} \times \\ &\times \delta N(\mathbf{r}_1)\delta N(\mathbf{r}_2) \frac{f}{|\mathbf{r}_1 - \mathbf{r}_2|}\frac{f}{|\mathbf{r}_2 - \mathbf{r}|} + \cdots\Bigg), \qquad (7.1)\end{aligned}$$

where μs are direction cosines ($\mu_i = (\mathbf{r}_i - \mathbf{r}_{i-1})/|\mathbf{r}_i - \mathbf{r}_{i-1}| \cdot \hat{\mathbf{k}}_0$). This equation tells that i) random fluctuations of scales much shorter than the mean free path linearize and disappear; ii) larger structure of scale a cause spreading of the wave to a cone of angular size $1/ka$; iii) a homogeneously distributed collection of fluctuations with range (a) sufficiently smaller then the free path (L) introduces a second order correction term for the refractive index of the order of $\sigma^2 a/L$, where σ^2 is the relative variance of fluctuations; iv) periodic structure of wavelength b can enhance scattering to side lobes at multiples of an angle $1/bk$ (times a cosine factor). For all our standard cases these effects are beyound all observation limits.

In another extreme one can perturb the refractive index. As simplest this causes for waves passed through different paths a phase shift. When the phase shifts are much smaller than unity, again a simple correction for the refractive index solves the problem; for scalar fluctuations even without observable trace. When the phase shifts are larger than unity ($\delta nak > 1$), each inhomogeneity spreads the wave individually to a cone δn, where δn is refractive index difference, and this spreading cumulates as the wave passes through more layers ($\sim \delta n\sqrt{z/a}$ for independent layers). Typical spreading is of the order of $10^{-10} \sim 10^{-20}$, and even the "worst case" estimate around 10^{-6}, meaning that no signatures can be observed nor do it cause any significant physical effets e.g. radiation pressure (proportional to spreading angle squared).

8. Non-diagonal matter effect

Let us now consider non-diagonal refractive indices. Within the standard model the matter potential is expected to be a matrix diagonal in the interaction basis of neutrinos. In any other basis the potential terms are generally non-diagonal. Moreover, some non-standard models include exotic interactions that generate non-diagonal components also in the weak basis [5].

It has been claimed that sizable non-diagonal components [19] could arise from a neutrino-neutrino scattering process $\nu_e(k)\nu_\mu(p) \to \nu_\mu(k)\nu_e(p)$. Allegedly the propagating neutrino changes its flavor with another neutrino that now acts as background. This suggestion has been adopted by several subsequent works [20], despite it has been strongly criticized [21].

However, the *scattering model* for the refraction discussed in this paper explicitly *excludes* such flavor changing terms. As concluded above, the change of a discrete state of the scatterer kills the coherence immediately. Hence the neutrino interchange processes can not generate any non-diagonal terms, as long as the background neutrinos can be treated as pure weak eigenstates.

Only when the background neutrinos are a coherent superposition of weak eigenstates, can the flavor interchange term reappear. For instance, assuming the scatterer to be in the state $\nu_m = \cos\theta_m \nu_e + \sin\theta_m \nu_\mu$, we should consider the process $\nu_e\nu_m \to \nu_\mu\nu_m$. This will now induce a non-zero contribution which is proportional to $1/2\sin 2\theta_m$. Here the angle θ_m refers only to the definition of the state, it equals with the effective matter mixing angle only in equilibrium. Note that the state $\cos\theta_m\nu_e - \sin\theta_m\nu_\mu$, orthogonal to the state ν_m contributes with the opposite sign as ν_m.

Because of the factor $1/2\sin 2\theta_m$ the off-diagonal components are usually very suppressed. The only case when they could have significance is a wide resonance that causes a sufficiently adiabatic transition. Particularly, this excludes a significant (adiabatic) neutrino flavor conversion caused primarily by the off-diagonal terms.

For similar reasons coherent spin-flip processes are impossible, whenever the spin is a well determined quantum number for the scatterers [22]. In such a case, any scattering process in the forward direction changing the spin of the scatteree would also change the spin of the scatterer, (because of the conservation of angular momentum) hence spoiling the coherence. The symmetry considerations also imply that the spin-flipping amplitudes vanish when the matter is completely unpolarized, or polarized to the direction of the neutrino momentum.

Consider now the matter effect in the mass basis. Hence the refractive terms appear between neutrinos in different mass eigenstates which corresponds to manifestly inelastic scattering processes. The momentum of the scattered neutrino state is then obtained from the equation

$$k(\theta) = \frac{k\left(M - \frac{\delta m^2}{k}\right) - \frac{\delta m^2}{k} M + \left(\frac{\delta m^2}{2k}\right)^2}{M - \frac{\delta m^2}{k} + k(1 - \cos\theta)}\,, \tag{8.1}$$

where it was assumed that $\delta m^2 \ll k^2$. For all relativistic neutrinos the shift of energy and momentum is practically ignorable, since all transformed neutrinos get the same shift, which is almost completely recoverable when they transform back to the original state. The irreversible energy loss is proportional to $(\delta m^2/2k^2)^2$, which is is too small to cause any observable effects.

This section can be summarized shortly, that the scattering pictures supports no non–diagonal terms in any significant amounts.

9. Polarized medium

Because of the inherent parity violation in weak interactions, the polarization of the medium changes the refractive effects of neutrinos [1, 4, 23] (see also [24]). As noted above, in unpolarized medium only the vector currents contribute to the matter effect, while the axial currents cancel. In polarized medium the axial currents give a contribution proportional to the average polarization ($P = \sum_s Y_s s$) in the direction of the neutrino trajectory.

The resulting potential in polarized (non-relativistic, non-degenerate) matter would be given by (in the scattering model as well as other models)

$$\begin{aligned} V_{\nu_e}(x) = \frac{G_F}{\sqrt{2}}\Big(&(1+4\xi)[N_{e^-}(x) - N_{e^+}(x)] + 2P_{e^-}N_{e^-}(x) + \\ &+ 2P_{e^+}N_{e^+}(x) + (1-4\xi)[N_{p^+}(x) - N_{p^-}(x)] + \\ &+ 2P_{p^+}N_{p^+}(x) + 2P_{p^-}N_{p^-}(x) - [N_n(x) - N_{\bar{n}}(x)] - \\ &- 2P_nN_n(x) - 2P_{\bar{n}}N_{\bar{n}}(x)\Big)\,, \end{aligned} \tag{9.1}$$

$$\begin{aligned} V_{\nu_\mu}(x) = \frac{G_F}{\sqrt{2}}\Big(&(-1+4\xi)[N_{e^-}(x) - N_{e^+}(x)] - 2P_{e^-}N_{e^-}(x) - \\ &- 2P_{e^+}N_{e^+}(x) + (1-4\xi)[N_{p^+}(x) - N_{p^-}(x)] + 2P_{p^+}N_{p^+}(x) + \\ &+ 2P_{p^-}N_{p^-}(x) - [N_n(x) - N_{\bar{n}}(x)] - \\ &- 2P_nN_n(x) - 2P_{\bar{n}}N_{\bar{n}}(x)\Big)\,. \end{aligned} \tag{9.2}$$

There is an important sign difference between polarized particles and antiparticles. When the particles and antiparticles are polarized oppositely, as in external magnetic field, the canonical signs are recovered, as predicted by the CP-symmetry.

The perpendicular polarization of matter could lead to interactions changing coherently the helicity of neutrinos. This is consistent with our requirement of the coherence, since the scatterer is not an eigenstate of J_z. In the standard electroweak model, however, all spin-flipping amplitudes of neutrinos are suppressed by a factor m_ν/E_ν [22], so that for these interactions to be significant, non-standard interactions coupling explicitly to right-handed states are required.

As an example we may consider the model [25] with an expanded scalar sector. The spin-flipping term would arise from the scattering of neutrinos from electrons by the exchange of the scalars, and it is given by

$$V_{\mathrm{SF}} \simeq \frac{hf}{4M^2} \sin \alpha P_e N_e \,, \tag{9.3}$$

where h and f are the Yukawa couplings between electrons and left- or right-handed neutrinos, respectively, and α is the mixing between the scalars. Since typically the polarization is due to external magnetic fields, the spin-flipping term may act like the magnetic moment term. To compare these terms, in the high temperature limit the spin-flipping potential term could be expressed as

$$V_{\mathrm{SF}} \simeq \mu_\nu B \left(\frac{N_e}{m_\ell m_e T} \right) , \tag{9.4}$$

where typically $\ell = \tau$. This is negligible in all the universe except in the core of the supernovae, where the spin-flipping potential may dominate the magnetic moment by no more than two orders of magnitudes.

Many authors have studied the passage of neutrinos in matter under magnetic field, obtaining the axial contribution by field theoretical method [26, 27]. These methods run the risk of hiding the physical phenomena under rather abstract methodology. It has been rediscovered that the field theoretical derivation equals the simple picture of polarized medium [28]. Recently the effects of tensor interactions in polarised matter were also studied [29].

The polarization of matter may influence the propagation and conversion of neutrinos in supernovae [23, 30]. It may be important also in the early universe [31, 27]. The early treatments of neutrino refraction [1, 4] also considered neutrinos passing through ferromagnets.

10. Degenerated medium

In the interiors of compact stars the many body effects may not be ignorable. In high densities the matter cannot be considered as a collection of independent particles, rather it should be modeled by a possibly degenerated Fermi gas.

Usually in the scattering theory the statistical effects are accounted for by including in the cross sections for each fermionic final state the Pauli blocking factor $1 - f(E)$. Here f is the Fermi-Dirac distribution function

$$f(E) = \frac{1}{1 + e^{(E-\mu)/T}}\,, \tag{10.1}$$

where μ is the chemical potential of the particle and E its final energy. For example, for the typical conditions in the core of a supernova the cross section of a process involving electrons may be reduced by as much as one order of magnitude, and also processes involving neutrons may be halved.

Following the adopted picture and postulates, we should take the blocking into account in calculating the refraction. In the scattering approach this is incorporated by including to the individual scattering amplitudes the respective blocking factors $1 - f(E)$ of the scatterers. The blocking factors then yield suppression factors for the refractive components, when summed over all the scatterers.

Obtaining the exact result for the refractive indices is quite cumbersome, since we have to deal with correlations among the scatterers, which is formally beyond the assumptions made in the derivation in previous sections. One could get an order of magnitude estimate of the blocking to the macroscopic potential using the blocking for average energies, $1 - f(\langle E \rangle)$. In the cores of supernovae and neutron stars it may vary from 0.1 to 0.9, in other environments the blocking is ignorable.

11. Conclusions

We have studied the passage of neutrinos through matter from a microscopic point of view. We have considered the scatterers as individual quantum degrees of freedom, instead of the usual approach of constant background. We have taken into account the recoil of the scatterer, thermal motion of the medium, and the interference of multiple scatterings. While not necessarily more fundamental than the other approaches, our method gives an essentially more physical way to understand the phenomena of particles propagating in a medium.

In our calculations we assumed the neutrinos to be in their mass shell, and the energy-momentum conservation law to be strictly in force. When necessary, we described the particles as wave packets, thus satisfying explicitly the uncertainty principle. To perform the calculations, the wave packets were assumed to have a gaussian form. We also assumed that they keep this form longitudinally during propagation and in the scattering processes. The final results may reflect the choice of our assumptions, but we believe that our picture is sufficiently well motivated to understand the phenomena.

We found that coherent interference effects can be modeled by an unsuppressed macroscopic locally constant potential, or a refractive index, if the following conditions are satisfied:

1. The scatterers are sufficiently independent of each others (including spin, position, momentum) at interaction length scales and not too dramatic non–plane parallel density variations exits.

2. The scattering is weak enough so that multiple scattering can be locally ignored: $f(0) \ll a\lambda$.

3. The wave packet describing the neutrinos is sufficiently long, $d \gg \lambda$.

4. The scattering does not change the quantum state of individual scatterers in any observable way.

5. Medium is non-degenerate.

Additional conditions may be posed for non-relativistic neutrinos.

The condition 1 is normally very valid. If it for some unknown reason failed, introduced effects would be little scintillation of the observations and minor momentum transfer to the scatterers.

The condition 3 is a direct consequence of the adopted wave packet formalism. Evidently it is satisfied in most environments. When it is not satisfied, there appears a suppression factor $\mathrm{O}(d/\lambda)$. The field theoretical results by definition ignore the localizable effects.

The condition 4 especially excludes flavor interchange processes between neutrinos and matter. This contradicts the previous claims of non-diagonal refractive elements induced by neutrino-neutrino interactions. Such components could be present in the hamiltonian only if the scatterers (neutrinos) are in a specific mixed state. In such a case the matter potential would be suppressed at least by the effective neutrino mixing in the medium. The spin-flipping refractive terms from standard model interactions are ignorable for all conceivable conditions.

Our model suggests that the matter effect is suppressed in a degenerate medium. This may affect neutrino conversions in supernovae and neutron stars where the matter is very degenerate. However, this effect is not supported by field theoretical studies. Given that the neutrino conversions are quite sensitive on the details of the refractive effects, this case requires a more detailed study.

Apart from the above exceptions, the scattering model gives results that are equivalent with those obtained by other methods. For most cases it is hence arbitrary to decide which method to prefer, but the scattering approach provides some benefits in certain cases. It would be most preferable for situations involving localizable effects, like very narrow controllable wave packets, or locally inhomogeneous medium, like a lattice. On the other hand, it may be less useful for studying systems where the mutual interactions generate high self energies beyond the conventional refraction phenomena. Neither of these extremes has much importance to neutrino physics.

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